## Math Virtual Learning

## Precalculus with Trigonometry

Students will solve basic trigonometric equations.

## May 6th, 2020

## Precalculus with Trigonometry Lesson: May 6th, 2020

## Objective/Learning Target:

Students will solve basic trigonometric equations.

## Let's Get Started! <br> Watch Video: Solving Trigonometric Equations

## Example: Find all possible values of $\theta$ so that $\cos \theta=\frac{1}{2}$.

You can solve this equation by connecting what you know about the cosine graph as well as the unit circle

$$
y=\cos x
$$



There are 3 ways to express your solution:

1. Degrees: $\square=600,3000$
2. Radians: $\square=\frac{\pi}{3}, \frac{5 \pi}{3}$
3. All solutions (because cosine is periodic)
 $\theta=\frac{\pi}{3}+2 \pi n, \theta=\frac{5 \pi}{3}+2 \pi n$, where n is an integer.

## Example: Solve $\sin x+\sqrt{2}=-\sin x$.

Begin by rewriting the equation so that $\sin x$ is isolated on one side of the equation.

$$
\begin{aligned}
\sin x+\sqrt{2} & =-\sin x & & \text { Write original equation. } \\
\sin x+\sin x+\sqrt{2} & =0 & & \text { Add } \sin x \text { to each side. } \\
\sin x+\sin x & =-\sqrt{2} & & \text { Subtract } \sqrt{2} \text { from each side. } \\
2 \sin x & =-\sqrt{2} & & \text { Combine like terms. } \\
\sin x & =-\frac{\sqrt{2}}{2} & & \text { Divide each side by } 2 . \\
x=\frac{5 \pi}{4}+2 n \pi & \text { and } & x=\frac{7 \pi}{4}+2 n \pi & \text { General solution }
\end{aligned}
$$

## Example: Solve $3 \tan ^{2} x-1=0$.

## Solution

Begin by rewriting the equation so that $\tan x$ is isolated on one side of the equation.

$$
\begin{aligned}
3 \tan ^{2} x-1 & =0 & & \text { Write original equation. } \\
3 \tan ^{2} x & =1 & & \text { Add } 1 \text { to each side. } \\
\tan ^{2} x & =\frac{1}{3} & & \text { Divide each side by } 3 . \\
\tan x & = \pm \frac{1}{\sqrt{3}}= \pm \frac{\sqrt{3}}{3} & & \text { Extract square roots. }
\end{aligned}
$$

Because $\tan x$ has a period of $\pi$, first find all solutions in the interval $[0, \pi)$. These solutions are $x=\pi / 6$ and $x=5 \pi / 6$. Finally, add multiples of $\pi$ to each of these solutions to get the general form

$$
x=\frac{\pi}{6}+n \pi \quad \text { and } \quad x=\frac{5 \pi}{6}+n \pi \quad \text { General solution }
$$

## Practice

Solve each equation for $0 \leq \theta<360$.

$$
\frac{\sqrt{3}}{2}=-\cos \theta \quad 3 \tan \theta=3 \sqrt{3}
$$

Solve each equation for $0 \leq \boldsymbol{\theta}<2 \boldsymbol{\pi}$.

$$
-8 \sin \theta=4 \sqrt{2}
$$

$$
4=5+\tan \theta
$$

Find all solutions to each equation in radians.

$$
\frac{2}{5} \cdot \cos \theta=\frac{\sqrt{2}}{5} \quad \frac{1}{2} \cdot \sin \theta=-\frac{\sqrt{3}}{4}
$$

## Practice - ANSWERS

$$
\begin{array}{lc}
\frac{\sqrt{3}}{2}=-\cos \theta & 3 \tan \theta=3 \sqrt{3} \\
\{150,210\} & \{60,240\} \\
-8 \sin \theta=4 \sqrt{2} & 4=5+\tan \theta \\
\left\{\frac{5 \pi}{4}, \frac{7 \pi}{4}\right\} & \left\{\frac{3 \pi}{4}, \frac{7 \pi}{4}\right\} \\
\frac{2}{5} \cdot \cos \theta=\frac{\sqrt{2}}{5} & \frac{1}{2} \cdot \sin \theta=-\frac{\sqrt{3}}{4} \\
\left(\frac{\pi}{4}+2 \pi n, \frac{7 \pi}{4}+2 \pi n\right\} & \left\{\frac{5 \pi}{3}+2 \pi n, \frac{4 \pi}{3}+2 \pi n\right\}
\end{array}
$$

## Additional Practice and Resources:

## Additional Resource Videos: Solving Trig Equations

# Solving Cosine Equation - Khan Academy 

## Additional Practice: Solving equations practice with answers

Solving basic trig equations practice

